Improving the accuracy of your SPICE simulations of powder cores by modelling inductor's variable saturation

Many users describe the inductance of their choke by a constant value in circuit simulations. For some core materials this is a suitable assumption, as the inductance remains constant until the saturation point of the choke is reached. Saturation starts at a specific current and the design may be some way off this; therefore, it is safe to assume inductance will not change.

Powder cores are different; even the smallest current can affect the inductance of the material. As a result the inductance is reliant on the current, and where the current changes so must the inductance. Many designers use an average value from the range their design will operate, which gives an acceptable degree of accuracy in circuit simulations; however, this is more of a challenge where there are large fluctuations in current.

The overall result can be identified when a design enters into the prototyping stage with physical hardware. Nevertheless, this requires additional time and resource, especially with designs that then fail.

In this paper we explain how the change in inductance from applied current can be described mathematically and be considered in a circuit simulation to improve design accuracy, based on the choke design or measurements.

Calculating the varying inductance of powder cores

The saturation effect of the powder cores is always described by the core manufacturer as a function of the permeability dependent to the applied magnetic field H. The index 'i' indicates initial values, i.e. no applied magnetic field in the core or no current in the inductor. As the permeability of the core is directly proportional to their $\mathrm{A_L}$value, the same equation can be used for this relation:

$$\frac{\mu_r}{\mu_{r,i}} = \frac{A_L}{A_{L,i}} = f(H) \tag{1}$$

The average magnetic path length l_m of the inductor cores is given in the datasheets or catalogues. This information can be merged with the number of turns N of the choke to get a mathematical description of the chokes saturation behaviour dependent to the applied current I. All we need now do is to replace the magnetic field strength by the mentioned values:

$$H = \frac{N \cdot I}{l_m} \tag{2}$$

Ignoring fringing effects, equations (1) and (2) can be used to describe the dependence of a choke's inductance to the applied current:

$$\boldsymbol{L(I) = L_i \cdot f\left(\frac{N \cdot I}{l_m}\right)} \quad (3)$$

This equation or its primitive can be used in different circuit simulation programs like SPICE. It has to be considered that the function given in equation (1) is only valid down to a specified drop of the initial permeability (often 30%) and does not consider fringing effects. If, due to this reason, the formula is not usable, it is also possible to measure the saturation behaviour of the choke, fit the measurement curve (i.e. with a simple stepwise function) and insert this function in the simulation program.

Example: Simulating the saturation effect in LT-SPICE

In LT-SPICE the saturation can be modelled using the flux statement. Here the user can enter the function of the total (linked) magnetic flux inside the coil depending on its current. The program will use Faraday's law to formulate the dependence between the inductor's current change and its voltage:

$$U_L = \frac{d(\psi)}{dt} = \frac{\partial(N \cdot \phi)}{\partial t} = L \cdot \frac{\partial I}{\partial t} \quad (4)$$

Based on Magnetics' catalogue (2015), this linked flux can be expressed by the following formula:

$$\psi(I) = L_i \cdot \left(a \cdot I + \frac{b}{2} \cdot \left(\frac{N}{l_m}\right) \cdot I^2 + \frac{c}{3} \cdot \left(\frac{N}{l_m}\right)^2 \cdot I^3 + \frac{d}{4} \cdot \left(\frac{N}{l_m}\right)^3 \cdot I^4 + \frac{e}{2} \cdot \left(\frac{N}{l_m}\right)^4 \cdot I^5\right) \quad (13)$$

See the appendix for how to derive this equation from the catalogue data.

Inserting this equation into SPICE, the direction of the flux has to be considered. So a 'negative' current will force a 'negative' flux, i.e. flux changes its direction. For this, positive and negative currents are defined by two logical statements, as shown in the following equation, which is inserted directly in the field of the inductance instead of a constant value.

```
flux={L0}*((({a}*x+0.5*{b}*{hi}*pwr(x,2)+{c}/3*pwr({hi},2)*pwr(x,3)+0.25*{d}*pwr({hi},3)
*pwr(x,4)+0.2*{e}*pwr({hi},4)*pwr(x,5))*(x>=0))-
(({a}*(-1)*x+0.5*{b}*{hi}*pwr((-1)*x,2)+{c}/3*pwr({hi},2)*pwr((-1)*x,3)
+0.25*{d}*pwr({hi},3)*pwr((-1)*x,4)+0.2*{e}*pwr({hi},4)*pwr((-1)*x,5))*(x<0)))
```

Example: Considering the core 77310-A7 (material KoolMµ $\mu_{r,i} = 125$) with N=57 turns. This will lead to a no load inductance of:

$$L_i = N^2 \cdot A_{L,i} = 57^2 \cdot 90nH = 292\mu H.$$

This value of the inductance at zero current and the constants values $a \ldots e$ from the Magnetics' catalogue (2015) are passed to SPICE via the parameter statement:

```
.params a=1  b=-9.918e-3 c=-5.044e-4 d=1.267e-5 e=-8.284e-8 hi=10.05 L0=292e-6
```

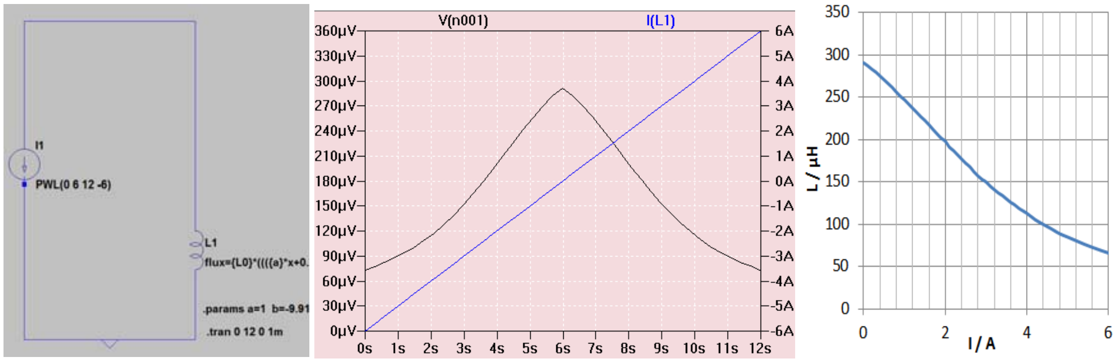

Figure 1: SPICE test simulation of saturating inductor: test circuit - inductor voltage and current diagram - measured saturation curve

In the above figure, the inductor is excited with an increasing current (blue line, right y-scale) with $\frac{\partial I}{\partial t} = 1\, {}^{A}/_{s}$. Excited like this, inductors voltage (black line, left y-scale) is similar to its inductivity, according to (1). It also shows that independent of the current direction (it changes its direction at t=6s, see the diagram), our inductor model shows the correct saturation curve.

The netlist file of this test circuit, as well as the SPICE simulation of a boost converter circuit, where this inductor model is used, can be downloaded as .asc or .net file. Please contact us at q-facts@acalbfi.com.

Appendix

To mathematically formulate the dependency of the inductors linked flux from its current, equation (1) has to be used. Assuming no saturation (μ_r=constant), the function of the flux in the core would be:

$$\phi = N \cdot I \cdot A_L = N \cdot I \cdot \frac{\mu_r \cdot \mu_0 \cdot A_{core}}{l_m} \quad (5)$$

Assuming saturation (μ_r drops with current), this formula changes to:

$$\partial\phi = N \cdot I \cdot A_L = N \cdot \frac{\mu_r \cdot \mu_0 \cdot A_{core}}{l_m} \cdot \partial I \quad (6)$$

The only current depending variable herein is μ_r, which can be replaced using equation (1). Integrating this formula will lead to the function of the flux showing a saturation effect with increasing current:

$$\partial\phi = N \cdot \frac{\mu_0 \cdot A_{core}}{l_m} \cdot \mu_{r,i} \cdot f(H) \cdot \partial I = N \cdot A_{L,i} \cdot f(H) \cdot \partial I \quad (7)$$

$$\phi(I) = N \cdot A_{L,i} \cdot \int f(H) \cdot \partial I \quad (8)$$

The given formula from Magnetics looks like the following. The values $a \ldots e$ are constant and given by the supplier:

$$\frac{\mu_r}{\mu_{r,i}} = f(H) = a + b \cdot H + c \cdot H^2 + d \cdot H^3 + e \cdot H^4 \quad (9)$$

Replacing the magnetic field in the inductor core and integrating this function will give the following:

$$f(I) = a + b \cdot \left(\frac{\mathrm{N}}{\mathrm{l_m}}\right) \cdot \mathrm{I} + c \cdot \left(\frac{\mathrm{N}}{\mathrm{l_m}}\right)^2 \cdot I^2 + d \cdot \left(\frac{\mathrm{N}}{\mathrm{l_m}}\right)^3 \cdot I^3 + e \cdot \left(\frac{\mathrm{N}}{\mathrm{l_m}}\right)^4 \cdot I^4 \quad (10)$$

$$F(I) = \int f(I)\, dI = a \cdot \mathrm{I} + \frac{b}{2} \cdot \left(\frac{\mathrm{N}}{\mathrm{l_m}}\right) \cdot I^2 + \frac{c}{3} \cdot \left(\frac{\mathrm{N}}{\mathrm{l_m}}\right)^2 \cdot I^3 + \frac{d}{4} \cdot \left(\frac{\mathrm{N}}{\mathrm{l_m}}\right)^3 \cdot I^4 + \frac{e}{2} \cdot \left(\frac{\mathrm{N}}{\mathrm{l_m}}\right)^4 \cdot I^5 + k \quad (11)$$

It is assumed that when $k = 0$, the magnetic flux is also zero when no current is applied.
Inserting this formula leads to the following formula of the flux:

$$\phi(I) = N \cdot A_{L,i} \cdot \left(a \cdot \mathrm{I} + \frac{b}{2} \cdot \left(\frac{\mathrm{N}}{\mathrm{l_m}}\right) \cdot I^2 + \frac{c}{3} \cdot \left(\frac{\mathrm{N}}{\mathrm{l_m}}\right)^2 \cdot I^3 + \frac{d}{4} \cdot \left(\frac{\mathrm{N}}{\mathrm{l_m}}\right)^3 \cdot I^4 + \frac{e}{2} \cdot \left(\frac{\mathrm{N}}{\mathrm{l_m}}\right)^4 \cdot I^5\right) \quad (12)$$

To get the linked flux, the whole equation needs to be multiplied by the number of turns as shown in equation (4). This leads to the final formula, which can be inserted in LT-SPICE.

$$\psi(I) = N^2 \cdot A_{L,i} \cdot \left(a \cdot \mathrm{I} + \frac{b}{2} \cdot \left(\frac{\mathrm{N}}{\mathrm{l_m}}\right) \cdot I^2 + \ldots\right) = L_i \cdot \left(a \cdot \mathrm{I} + \frac{b}{2} \cdot \left(\frac{\mathrm{N}}{\mathrm{l_m}}\right) \cdot I^2 + \ldots\right) \quad (13)$$

The same procedure can be applied on any function that describes the saturation curve of the inductance.